



Cambridge International AS & A Level

CANDIDATE
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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2024

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



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- 1 (a) Sketch the graph of $y = |x - 2a|$, where a is a positive constant. [1]

- (b) Solve the inequality $2x - 3a < |x - 2a|$. [2]

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2 Express $\frac{6x^2-9x-16}{2x^2-5x-12}$ in partial fractions.

[5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 3 The variables x and y satisfy the equation $a^{2y-1} = b^{x-y}$, where a and b are constants.

(a) Show that the graph of y against x is a straight line. [3]

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(b) Given that $a = b^3$, state the equation of the straight line in the form $y = px + q$, where p and q are rational numbers in their simplest form. [2]

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4 The equation of a curve is $ye^{2x} + y^2e^x = 6$.

Find the gradient of the curve at the point where $y = 1$.

[6]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The background is white, and there are no margins or other markings present.

- 5 (a) It is given that the equation $e^{2x} = 5 + \cos 3x$ has only one root.

Show by calculation that this root lies in the interval $0.7 < x < 0.8$. [2]

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- (b) Show that if a sequence of values in the interval $0.7 < x < 0.8$ given by the iterative formula

$$x_{n+1} = \frac{1}{2} \ln(5 + \cos 3x_n)$$

converges then it converges to the root of the equation in part (a). [1]

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- (c) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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(b) Find the exact value of $\int_0^{\frac{2}{a}} x e^{-ax} dx$.

[5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

7 (a) Show that $\cos^4 \theta - \sin^4 \theta \equiv \cos 2\theta$.

[3]

[illegible]

(b) Hence find the exact value of $\int_{-\frac{1}{8}\pi}^{\frac{1}{8}\pi} (\cos^4 \theta - \sin^4 \theta + 4 \sin^2 \theta \cos^2 \theta) d\theta$. [6]

[illegible]

- 8** The points A , B and C have position vectors $\overrightarrow{OA} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, where O is the origin. The line l_1 passes through B and C .

(a) Find a vector equation for l_1 . [3]

[illegible]

The line l_2 has equation $\mathbf{r} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$.

(b) Find the coordinates of the point of intersection of l_1 and l_2 . [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (c) The point D on l_2 is such that $AB = BD$.

Find the position vector of D .

[5]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

9 The complex numbers z and ω are defined by $z = 1 - i$ and $\omega = -3 + 3\sqrt{3}i$.

- (a) Express $z\omega$ in the form $a + bi$, where a and b are real and in exact surd form. [1]

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- (b) Express z and ω in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the exact values of r and θ in each case. [4]

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- (c) On an Argand diagram, the points representing ω and $z\omega$ are A and B respectively.

Prove that OAB is an isosceles right-angled triangle, where O is the origin. [2]

- (d) Using your answers to part (b), prove that $\tan \frac{5}{12}\pi = \frac{\sqrt{3}+1}{\sqrt{3}-1}$. [3]

- 10 (a) By writing $y = \sec^3 \theta$ as $\frac{1}{\cos^3 \theta}$, show that $\frac{dy}{d\theta} = 3 \sin \theta \sec^4 \theta$. [2]

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- (b) The variables x and θ satisfy the differential equation

$$(x^2 + 9) \sin \theta \frac{d\theta}{dx} = (x + 3) \cos^4 \theta.$$

It is given that $x = 3$ when $\theta = \frac{1}{3}\pi$.

Solve the differential equation to find the value of $\cos \theta$ when $x = 0$. Give your answer correct to 3 significant figures. [8]

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[illegible]

Additional page

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[illegible]

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